

QUEEN'S COLLEGE, CORK.

HONOR EXAMINATION, JUNIOR CLASS, JUNE 1ST, 1859

1. Prove that $x^m \times x^n = x^{m+n}$ for all values of the indices m and n , positive, negative, integral, and fractional.

2. Simplify $\left\{ \frac{x + \sqrt{x^2 - 1}}{x - \sqrt{x^2 - 1}} - \frac{x - \sqrt{x^2 - 1}}{x + \sqrt{x^2 - 1}} \right\} \div \frac{x^2 - 1}{\sqrt{x^2 + 1}}$.

3. Prove the rule for G C M. Find the G C M of $6x^4 - 25x^2 - 9$ and $3x^3 - 15x^2 - 5$. Also of $x^3 + y^3 + z^3 - 3xyz$ and $x^2 + 2xy + y^2 - z^2$.

4. Solve the equations $(x + 5)^{\frac{1}{m}} = (x^2 + 40x + 16)^{\frac{1}{2m}}$
 $x^2 - x + b = (x^2 - x - b^2)^{\frac{1}{2}}$

5. Solve the simultaneous equations—

$$\begin{aligned} x + y + z &= 6 \\ x - 2y + x &= 0 \quad \text{and} \quad \begin{aligned} x^2 + xy + 2y^2 &= 74 \\ x^2 - xy + 3y^2 &= 69 \end{aligned} \\ 3x - y + 5z &= 16 \end{aligned}$$

6. Explain the genesis of equations of the higher orders from simple equations. Construct a quadratic whose roots shall be $3 + \sqrt{5}$ and $3 - \sqrt{5}$; also a cubic whose roots shall be 0, 2, and 3.

7. Prove that in a geometric progression the product an any two terms equidistant from a given term is always the same. What is the corresponding theorem in arithmetical progression?

8. The first term of an arithmetical series is $2\frac{1}{2}$, the last $55\frac{1}{2}$, the number of terms = 100; find the sum of the series.

9. Extract the cube root of .7854 to three places of decimals.

10. Given two sides of a triangle and the included angle, deduce formulæ for solving the triangle.

11. Prove the following formulæ of plane trigonometry, viz.:—

$$\begin{aligned} \tan(180^\circ + A) &= \tan A \\ \cos(180^\circ - A) &= -\cos A \\ 1 + \cos(A + B) \cos(A - B) &= \cos^2 A + \cos^2 B \\ \cot A &= \cot 2A \pm \sqrt{1 + \cot^2 2A} \end{aligned}$$

12. Shew *a priori* that a formula expressing $\frac{A}{2}$ in terms of $\sin A$ ought to have four values; while one expressing $\tan \frac{A}{2}$ in terms of $\tan A$ ought to have two values.

13. Shew how to find the distance between two inaccessible points—1st, when in the horizontal plane; 2ndly, when not in that plane.

14. If a circle roll within another of twice its diameter, any point in the circumference of the former will describe a straight line.

15. Explain Euclid's doctrine of ratio and of proportion; shew that his definition of ratio is imperfect unless accompanied by his definition of proportion. Does Euclid's definition of proportion furnish a direct and immediate test for determining whether four magnitudes are proportional or not?

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